### F07AJF (SGETRI/DGETRI) - NAG Fortran Library Routine Document

Note. Before using this routine, please read the Users' Note for your implementation to check the interpretation of bold italicised terms and other implementation-dependent details.

## 1 Purpose

F07AJF (SGETRI/DGETRI) computes the inverse of a real matrix A, where A has been factorized by F07ADF (SGETRF/DGETRF).

# 2 Specification

```
SUBROUTINE FO7AJF(N, A, LDA, IPIV, WORK, LWORK, INFO) ENTRY sgetri(N, A, LDA, IPIV, WORK, LWORK, INFO) INTEGER N, LDA, IPIV(*), LWORK, INFO real A(LDA,*), WORK(LWORK)
```

The ENTRY statement enables the routine to be called by its LAPACK name.

# 3 Description

To compute the inverse of a real matrix A, the routine must be preceded by a call to F07ADF (SGETRF/DGETRF), which computes the LU factorization of A as A = PLU. The inverse of A is computed by forming  $U^{-1}$  and then solving the equation  $XPL = U^{-1}$  for X.

#### 4 References

[1] Du Croz J J and Higham N J (1992) Stability of methods for matrix inversion *IMA J. Numer.* Anal. 12 1–19

### 5 Parameters

1: N — INTEGER

On entry: n, the order of the matrix A.

Constraint:  $N \geq 0$ .

2: A(LDA,\*) - real array

Input/Output

**Note:** the second dimension of the array A must be at least max(1,N).

On entry: the LU factorization of A, as returned by F07ADF (SGETRF/DGETRF).

On exit: the factorization is overwritten by the n by n matrix  $A^{-1}$ .

3: LDA — INTEGER Input

On entry: the first dimension of the array A as declared in the (sub)program from which F07AJF (SGETRI/DGETRI) is called.

Constraint: LDA  $\geq \max(1,N)$ .

4: IPIV(\*) — INTEGER array

Input

**Note:** the dimension of the array IPIV must be at least max(1,N).

On entry: the pivot indices, as returned by F07ADF (SGETRF/DGETRF).

5: WORK(LWORK) — *real* array

Work space

On exit: if INFO = 0, WORK(1) contains the minimum value of LWORK required for optimum performance.

#### **6:** LWORK — INTEGER

Input

On entry: the dimension of the array WORK as declared in the (sub)program from which F07AJF (SGETRI/DGETRI) is called.

Suggested value: for optimum performance LWORK should be at least  $N \times nb$ , where nb is the **blocksize**.

Constraint: LWORK  $\geq N$ .

**7:** INFO — INTEGER

Output

On exit: INFO = 0 unless the routine detects an error (see Section 6).

### 6 Error Indicators and Warnings

INFO < 0

If INFO = -i, the *i*th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If INFO = i, the ith diagonal element of the factor U is zero, U is singular, and the inverse of A cannot be computed.

# 7 Accuracy

The computed inverse X satisfies a bound of the form:

$$|XA - I| \le c(n)\epsilon |X|P|L||U|,$$

where c(n) is a modest linear function of n, and  $\epsilon$  is the **machine precision**.

**Note.** A similar bound for |AX - I| cannot be guaranteed, although it is almost always satisfied. See Du Croz and Higham [1].

### 8 Further Comments

The total number of floating-point operations is approximately  $\frac{4}{3}n^3$ .

The complex analogue of this routine is F07AWF (CGETRI/ZGETRI).

# 9 Example

To compute the inverse of the matrix A, where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by F07ADF (SGETRF/DGETRF).

### 9.1 Program Text

**Note.** The listing of the example program presented below uses bold italicised terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
FO7AJF Example Program Text
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.. Parameters ..
INTEGER
                NIN, NOUT
PARAMETER.
                (NIN=5,NOUT=6)
                NMAX, LDA, LWORK
INTEGER
PARAMETER
               (NMAX=8,LDA=NMAX,LWORK=64*NMAX)
.. Local Scalars ..
INTEGER
                I, IFAIL, INFO, J, N
.. Local Arrays ..
real
                A(LDA,NMAX), WORK(LWORK)
INTEGER
                 IPIV(NMAX)
.. External Subroutines ..
EXTERNAL
                sgetrf,\ sgetri,\ 	exttt{XO4CAF}
.. Executable Statements ..
WRITE (NOUT,*) 'F07AJF Example Program Results'
Skip heading in data file
READ (NIN,*)
READ (NIN,*) N
IF (N.LE.NMAX) THEN
   Read A from data file
   READ (NIN,*) ((A(I,J),J=1,N),I=1,N)
   Factorize A
   CALL sgetrf(N,N,A,LDA,IPIV,INFO)
   WRITE (NOUT,*)
   IF (INFO.EQ.O) THEN
      Compute inverse of A
      CALL sgetri(N,A,LDA,IPIV,WORK,LWORK,INFO)
      Print inverse
      IFAIL = 0
      CALL XO4CAF('General',' ',N,N,A,LDA,'Inverse',IFAIL)
   ELSE
      WRITE (NOUT,*) 'The factor U is singular'
   END IF
END IF
STOP
END
```

# 9.2 Program Data

	m Data	Progra	Example	F07AJF
:Value of N				4
	-0.89	2.05	2.88	1.80
	-3.80	-0.95	-2.95	5.25
	-1.04	-2.90	-2.69	1.58
:End of matrix A	0.80	-0.59	-0.66	-1.11

#### Program Results 9.3

FO7AJF Example Program Results

Inverse								
	1	2	3	4				
1	1.7720	0.5757	0.0843	4.8155				
2	-0.1175	-0.4456	0.4114	-1.7126				
3	0.1799	0.4527	-0.6676	1.4824				
4	2.4944	0.7650	-0.0360	7.6119				